



# Fractal Properties of the Thyrotropic Feedback Control

## Implications of a Nonlinear Model Compared with Empirical Data

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# The Thyrotropic Feedback Control

## Is it really well known?



### 70 Years of Investigation on the Pituitary Thyroid Feedback Control:

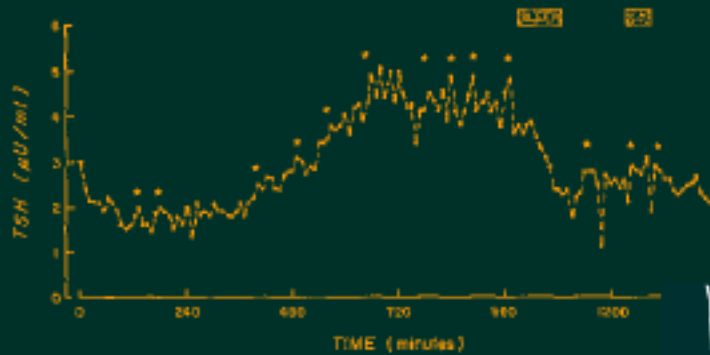
- Many new insights on molecular and cellular level.
- Poor understanding of the system in its whole.
- A bunch of different and contradictory models.
- Still open questions.



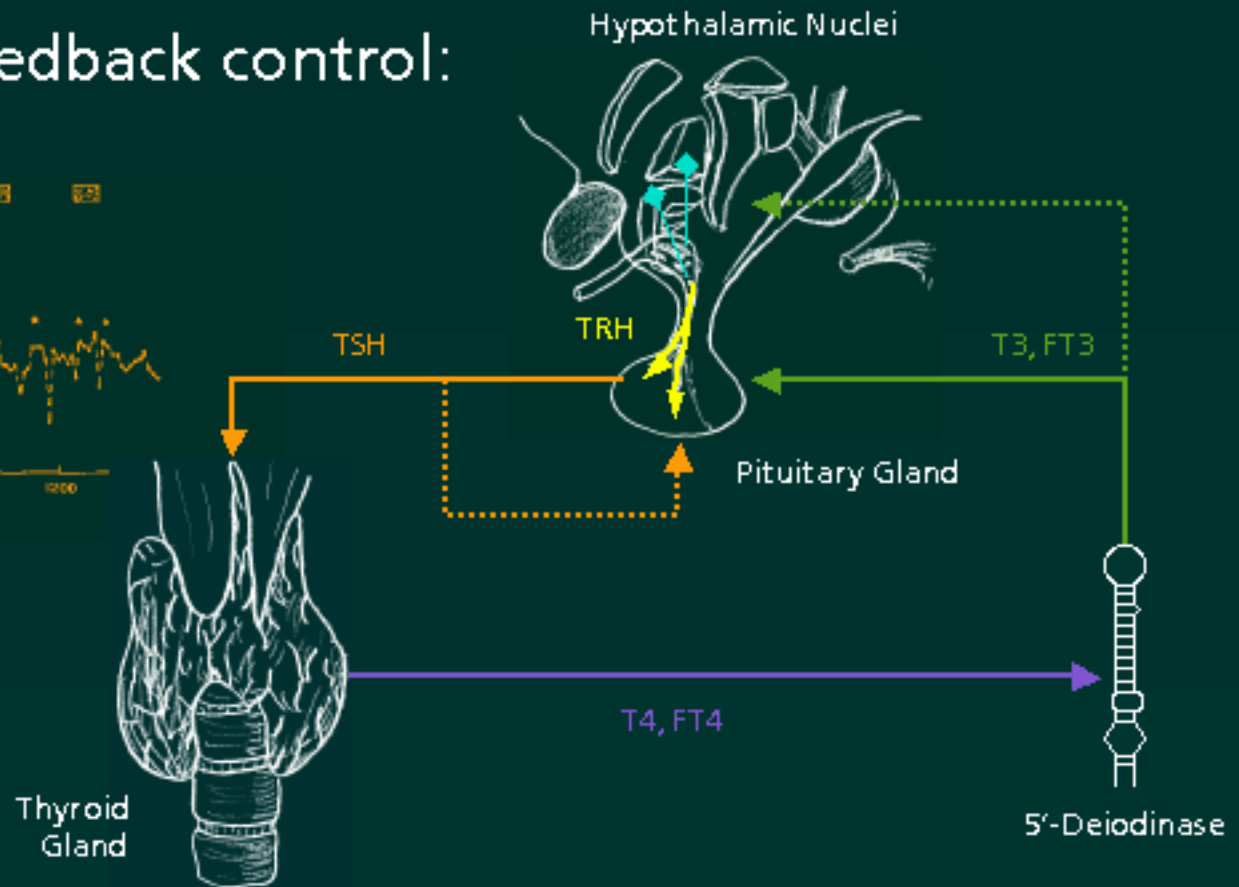
# The Thyrotropic Feedback Control Overview



Overview of feedback control:



[Greenspan 1986]



Mechanisms yielding TSH pulses are still unknown.



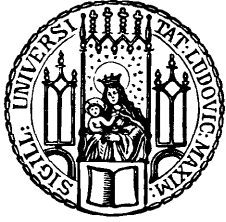
# The Thyrotropic Feedback Control Model based inference



Human pituitary **in vivo** practically inaccessible  
for research.

Does a simulative approach provide new insights  
into the information processing structure?

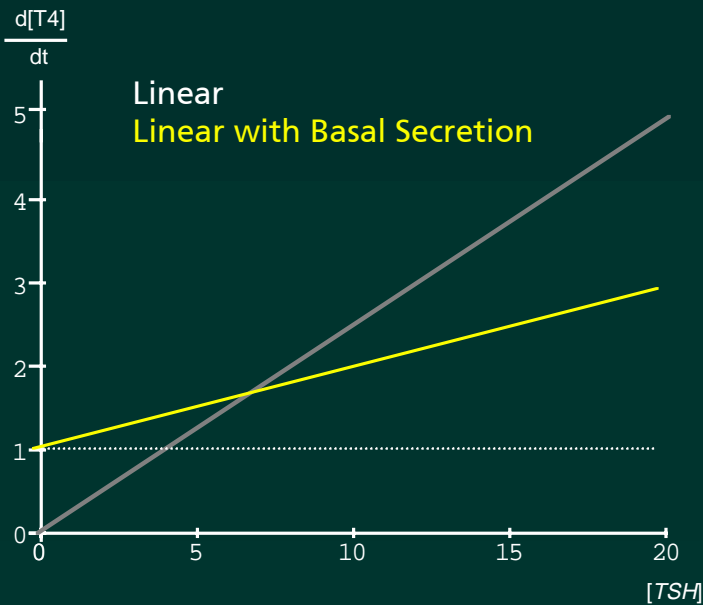




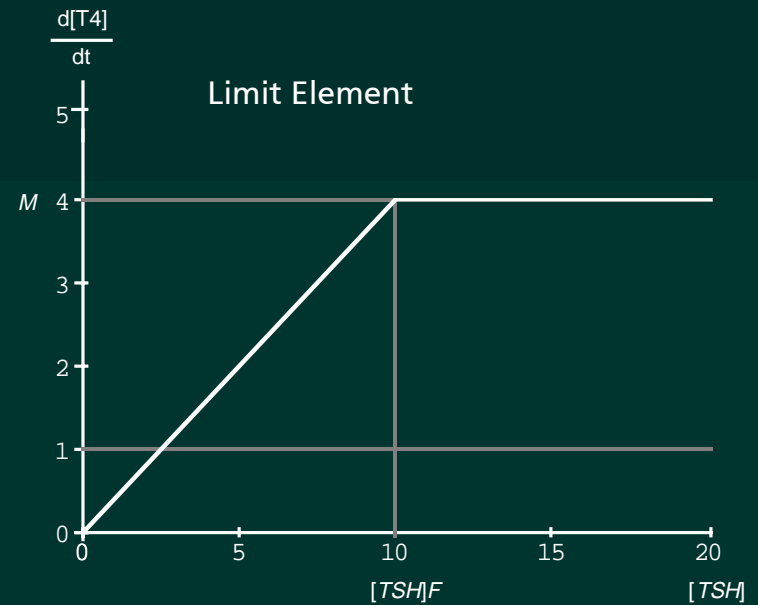
# The Thyrotropic Feedback Control Different Models



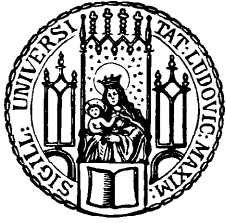
“Behaviourally isomorphic” models:



Danziger and Elmergreen 1956  
Norwich and Reiter 1965  
Roston 1959  
DiStefano 1969



Wilkin et al. 1977

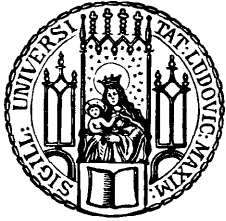


# The Thyrotropic Feedback Control A New Model (I)



## Requirements for a new approach:

- Empirically based transfer characteristics
- Parametrically isomorphic information processing structure
- Modelling of complete feedback loop

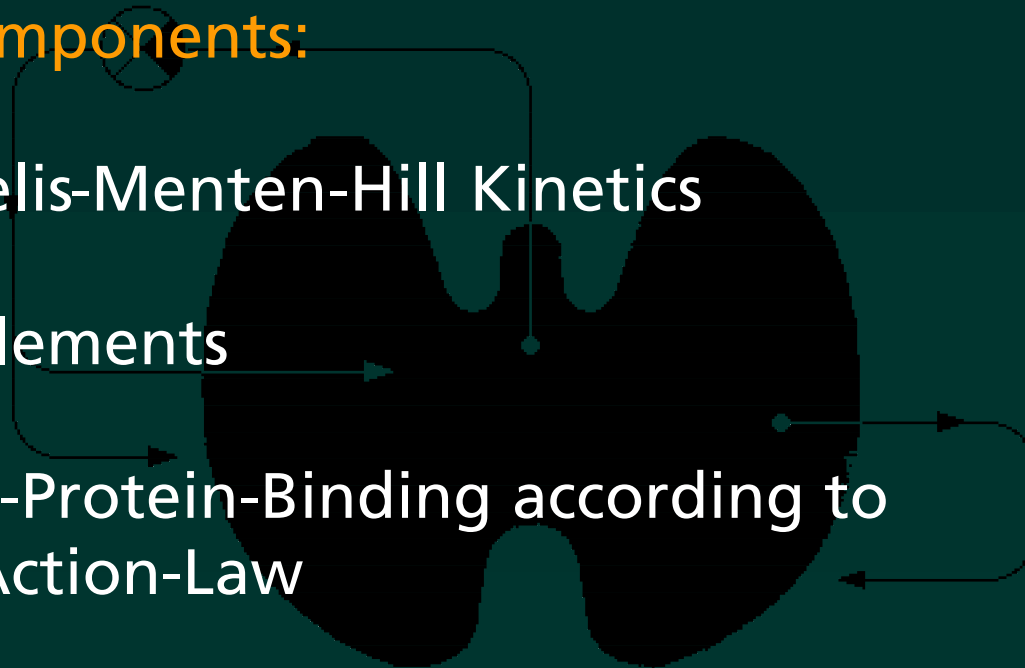


# The Thyrotropic Feedback Control A New Model (II)



## Main Components:

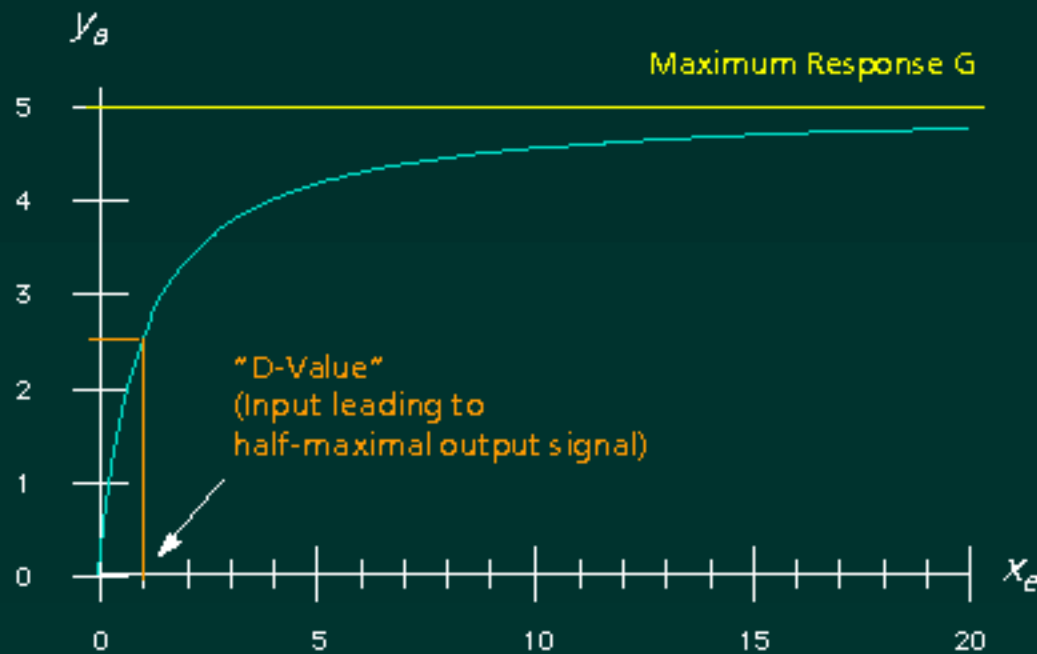
- Michaelis-Menten-Hill Kinetics
- ASIA-Elements
- Plasma-Protein-Binding according to Mass-Action-Law
- Non competitive quotient inhibition (NCQI)





# The Thyrotropic Feedback Control

## Michaelis Menten Hill Kinetics

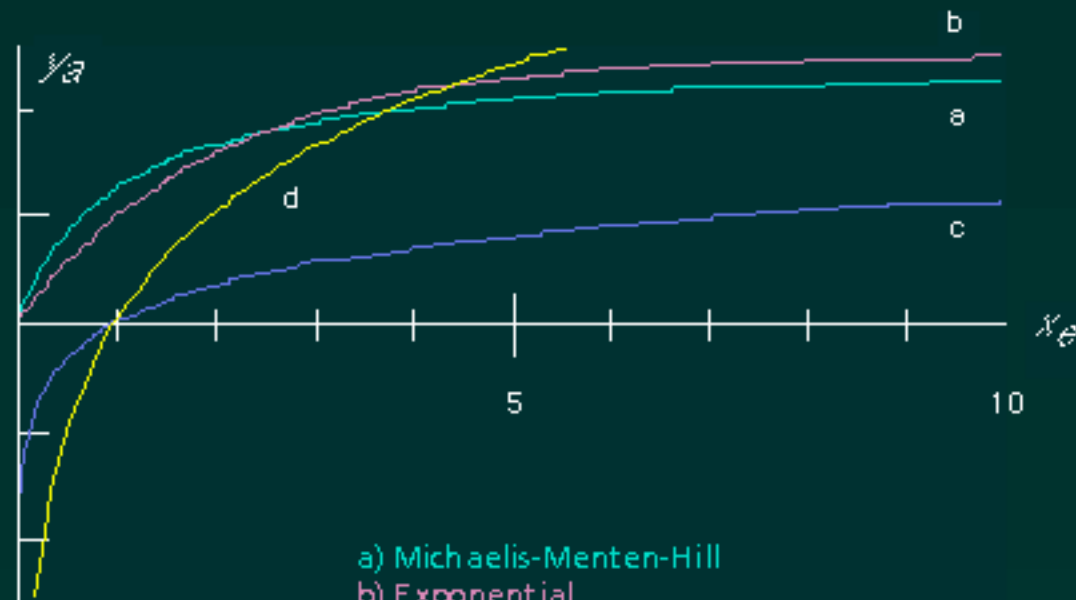


$$y_a = \frac{Gx_e}{D + x_e}$$

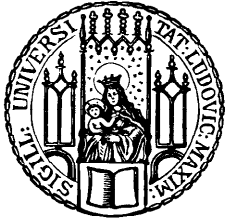




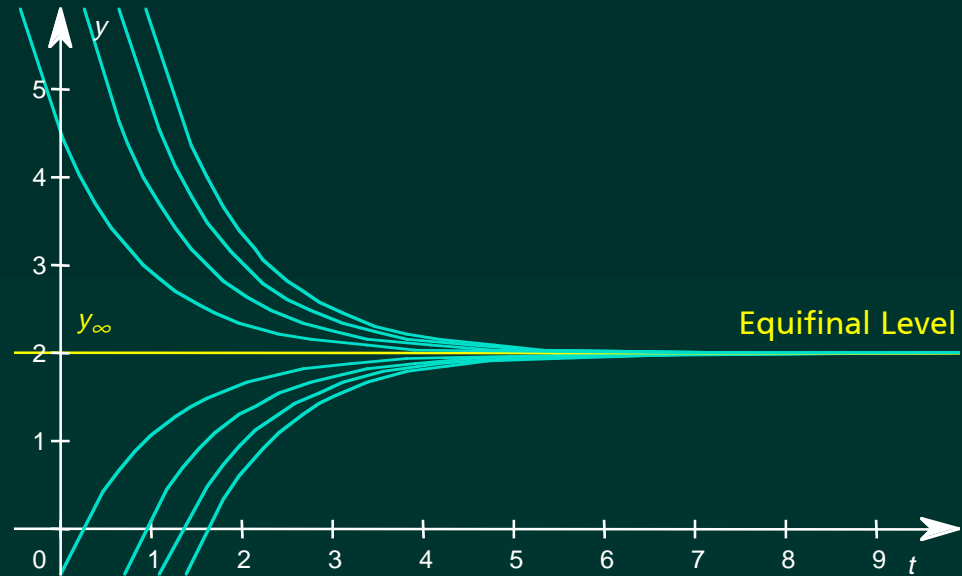
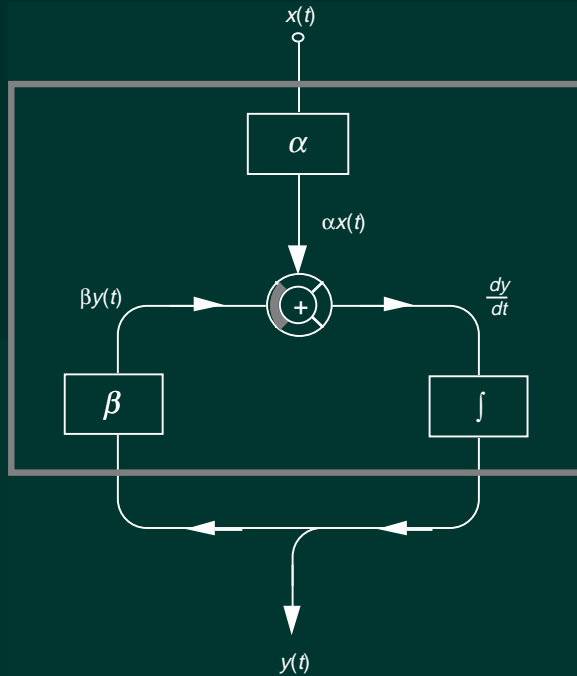
# The Thyrotropic Feedback Control Different Nonlinear Models



- a) Michaelis-Menten-Hill
- b) Exponential
- c) Logarithmic
- d) Logarithmic,  
adapted for  $x_e=4$



# The Thyrotropic Feedback Control ASIA Element



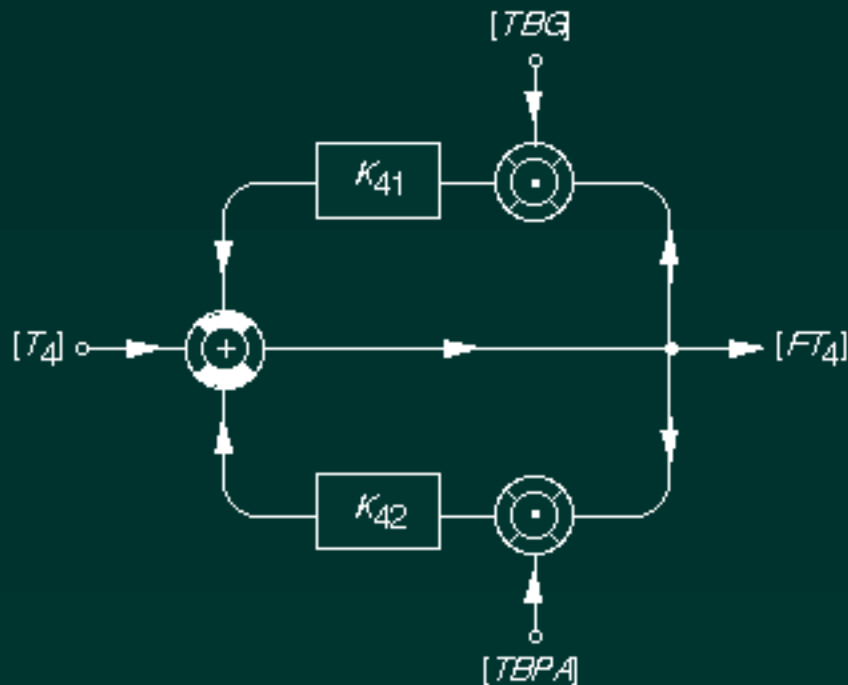
$$\frac{dy}{dt} = \alpha x(t) - \beta y(t)$$

$$y_{\infty} = \frac{\alpha x(t)}{\beta}$$

$$\tau_1 = \frac{1}{\beta}$$



# The Thyrotropic Feedback Control Plasma Protein Binding



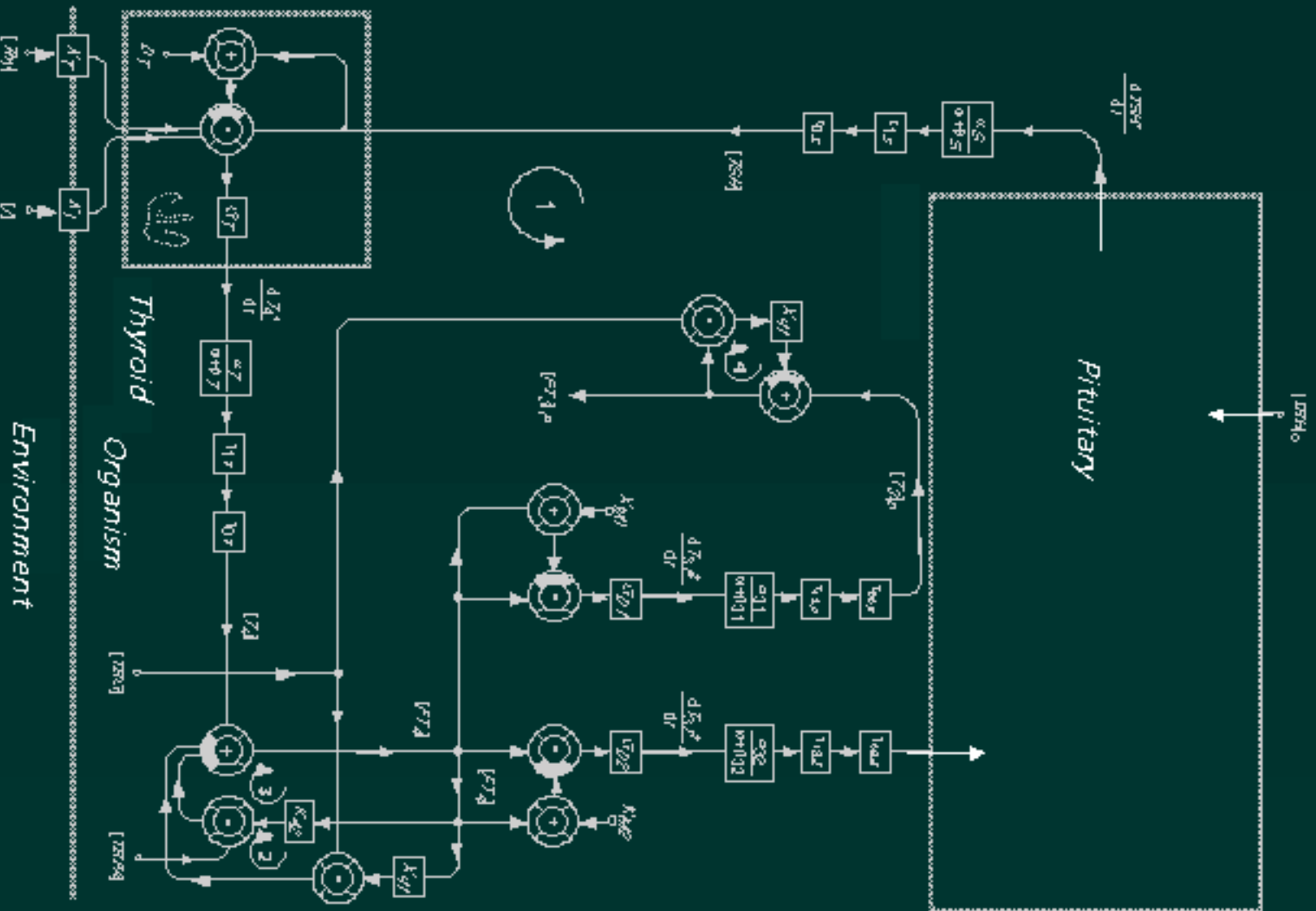
$$[BT_4] = K_{41}FT_4[TBG] + K_{42}FT_4[TBPA]$$

$$[FT_4] = [T_4] - [BT_4]$$

$$[FT_4] = \frac{[T_4]}{1 + K_{41}[TBG] + K_{42}[TBPA]}$$

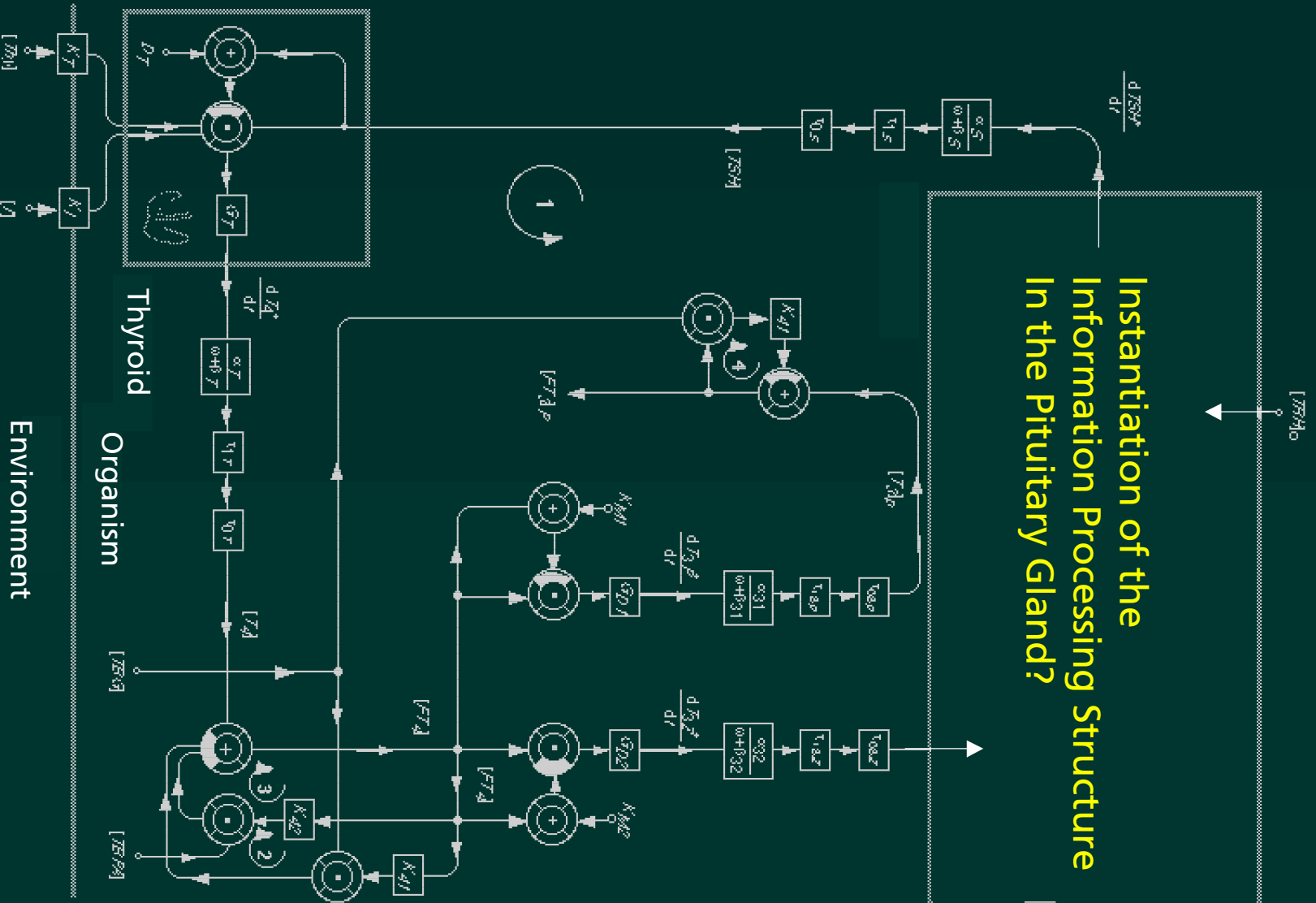
$$[FT_3]_P = \frac{[T_3]_P}{1 + K_{30}[TBG]}$$

# The Thyrotropic Feedback Control Universal Model



# The Thyrotropic Feedback Control Universal Model

Instantiation of the  
Information Processing Structure  
In the Pituitary Gland?

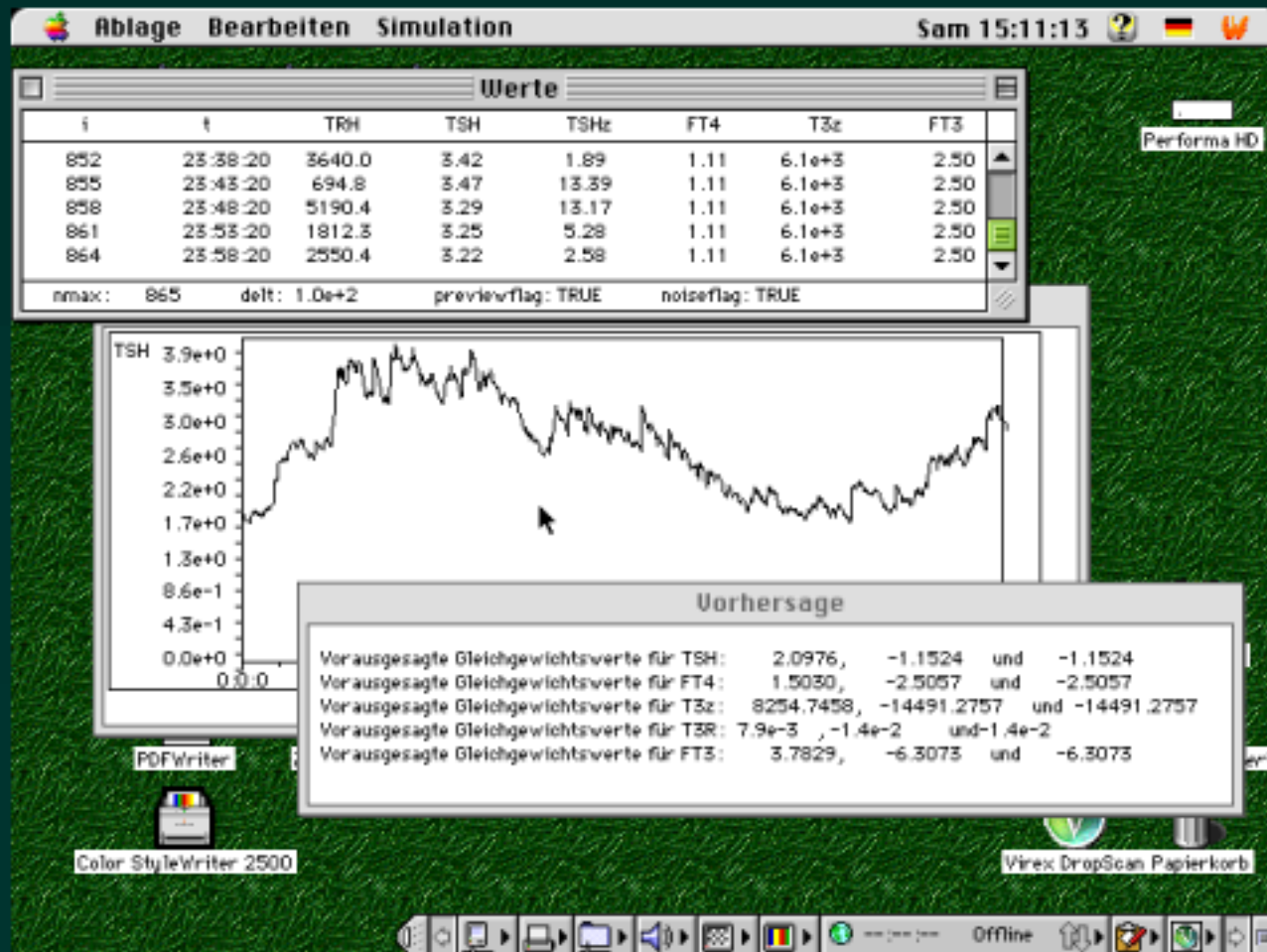








# The Thyrotropic Feedback Control Computer Simulations





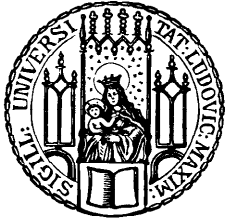


# The Thyrotropic Feedback Control Comparing Different Models



## Methods for characterization:

- Equifinal equilibrium hormone levels
- Qualitative inspection of signal course
- Fractal analysis of time series

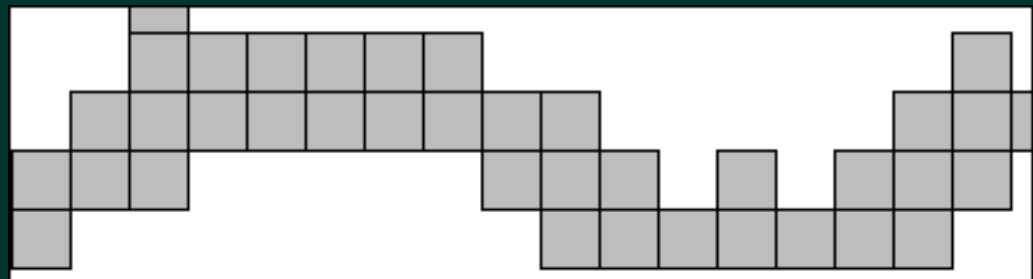
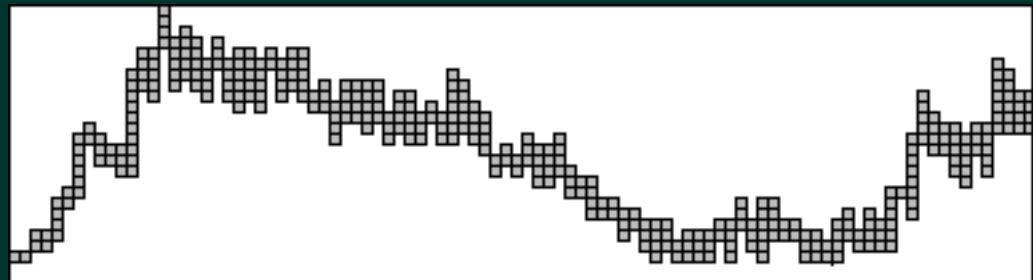
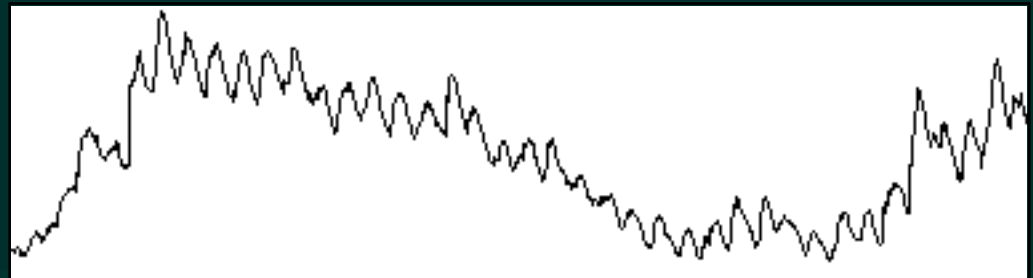


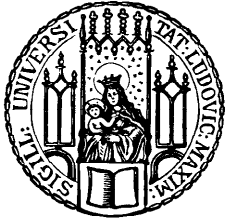
# The Thyrotropic Feedback Control Fractal Dimensioning (I)



Capacity dimension:

$$D_0 = \lim_{s \rightarrow 0} \frac{\log N(s)}{\log \frac{1}{s}}$$





# The Thyrotropic Feedback Control Fractal Dimensioning (II)



## Correlation dimension:

$$\vec{x}_i = (x_i, x_{i+p}, x_{i+2p}, \dots, x_{i+(m-1)p})$$

$$C(\varepsilon) = \frac{1}{M^2} \sum_{\substack{i,j=1 \\ i \neq j}}^M u_0(\varepsilon - ||\vec{x}_j - \vec{x}_i||)$$

$$n_i(\varepsilon) = \frac{1}{N} \sum_{j=1}^N u_0(\varepsilon - ||\vec{x}_j - \vec{x}_i||)$$

$$D_2 = \lim_{\varepsilon \rightarrow 0} \frac{\log C(\varepsilon)}{\log \varepsilon}$$

$$u_0(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases}$$



# The Thyrotropic Feedback Control Results (I)



## Results

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# The Thyrotropic Feedback Control Results (I)



## Analytic solutions for Equilibrium levels:

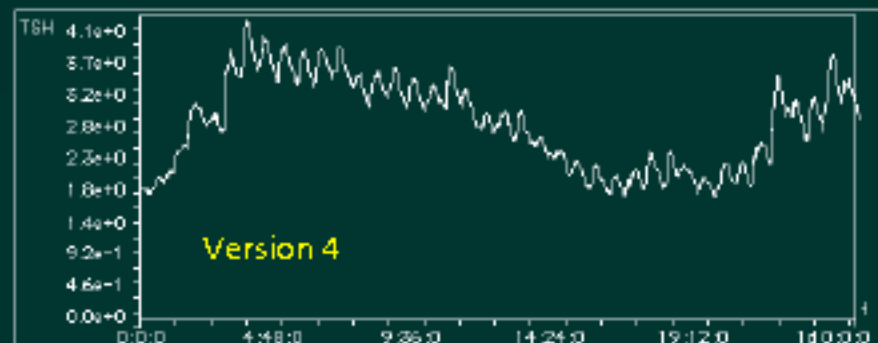
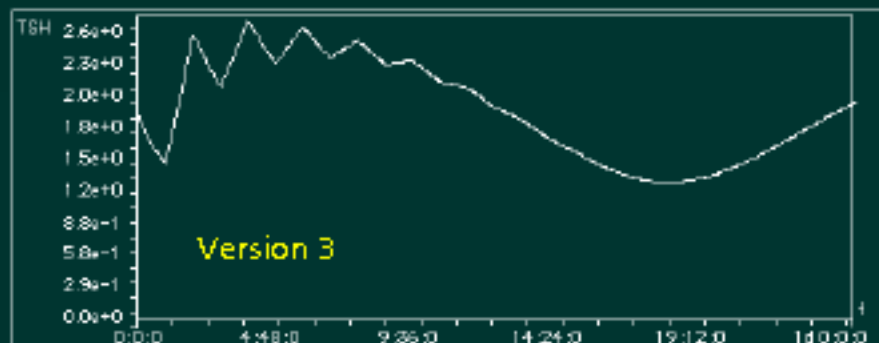
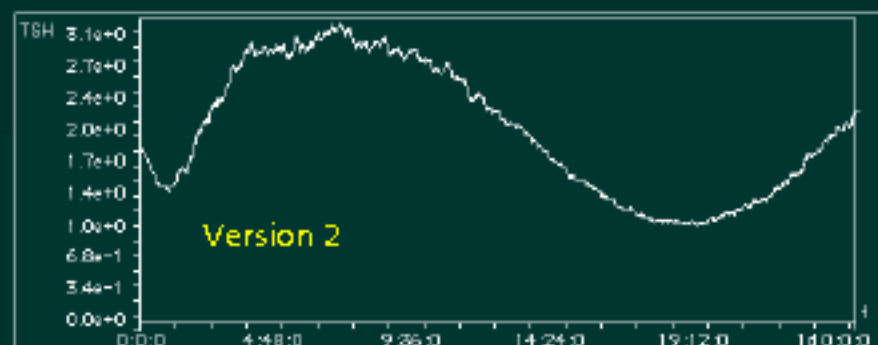
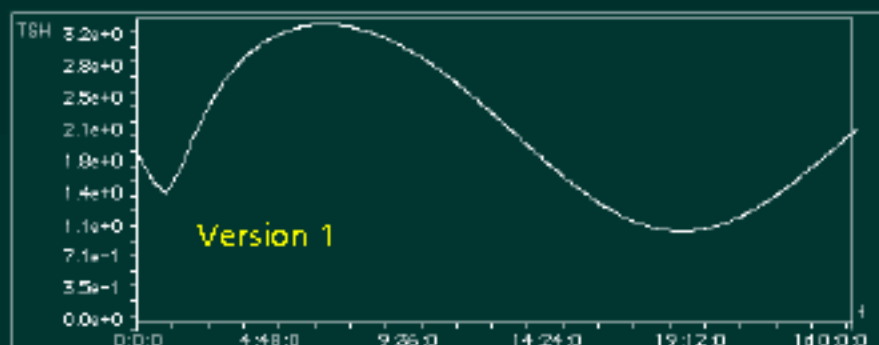
Parameter	Solution 1	Solution 2	Solution 3
TSH	1.8 mU/l	-1.2 mu/l	-1.2 mu/l
FT4	1.4 ng/dl	-2.4 ng/dl	-2.4 ng/dl
FT3	3.5 pg/ml	-6.0 pg/ml	-6.0 pg/ml

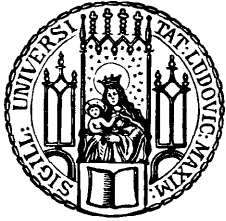


# The Thyrotropic Feedback Control Results (II)



## Simulated Time Series:





# The Thyrotropic Feedback Control Results (III)



## Fractal Dimensions:

Dimension (Mean)	$D_0$	$D_2$	m
Empirical	1.20	1.75	19.63
Model 1	0.96**	0.76 **	1.00 **
Model 2	1.04 **	0.77 **	1.00 **
Model 3	0.99 **	0.74 **	1.13 **
Model 4	1.18	1.91	20.14

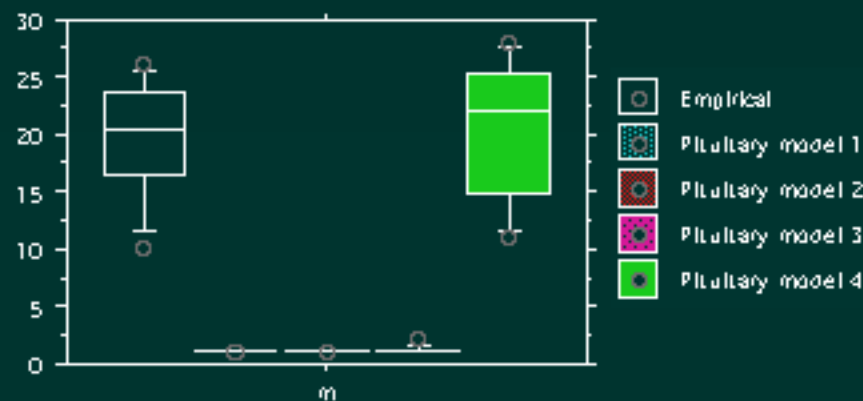
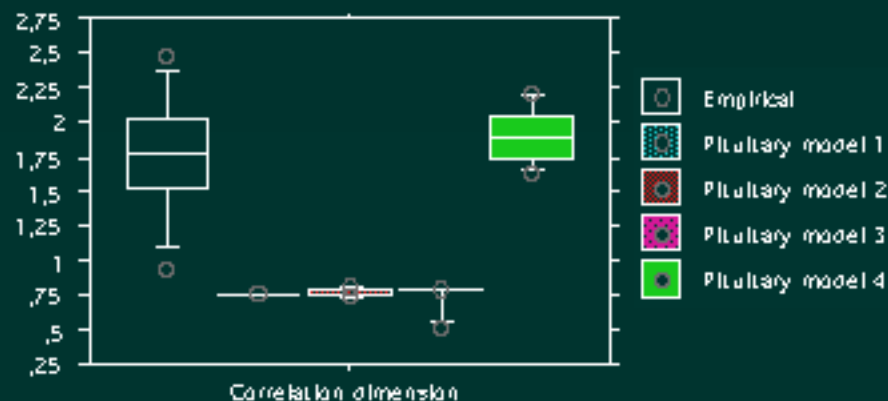
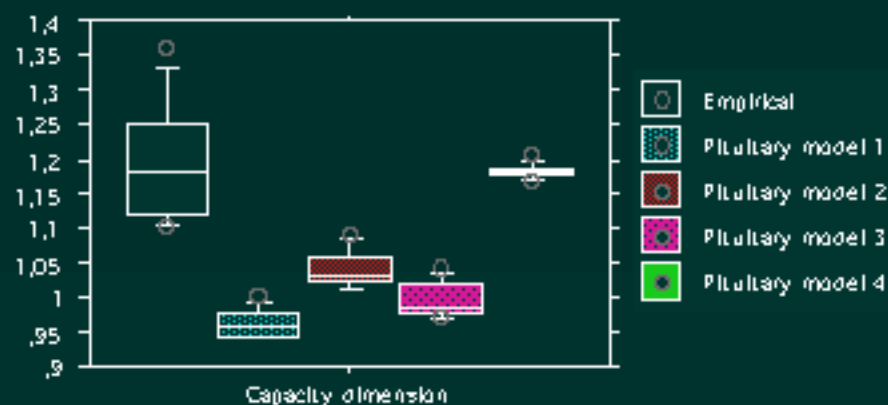
\*\* :  $p < 0.001$



# The Thyrotropic Feedback Control Results (IV)

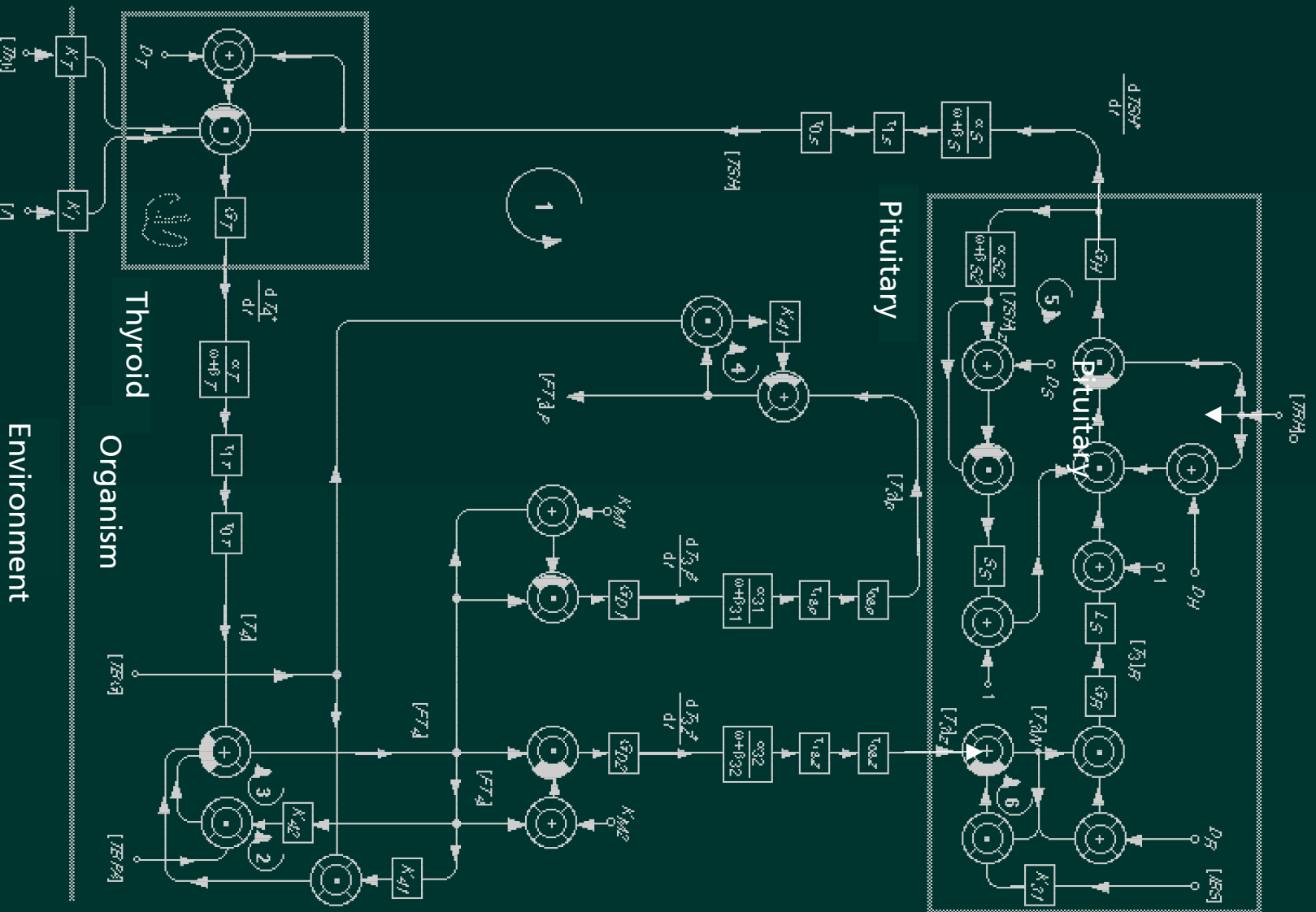


## Fractal Dimensions:





# The Thyrotropic Feedback Control Possible Complete Model







# The Thyrotropic Feedback Control Conclusion



- Non competitive quotient inhibition (NCQI) acts as sufficient explanation for emergence of equifinal levels in both healthy and diseased state.
- Simple NCQI fails to deliver plausible time series.
- Combination of NCQI with noise and ultra-short feedback of TSH might be an explanation for the dynamics of nature.



# The Thyrotropic Feedback Control Additional Information



Point your browser to:

<http://spina.medical-cybernetics.de>

or

<http://link.medinn.med.uni-muenchen.de/spina/>